III. "Solving" Time-dependent Schrödinger Equation Formally Without Specifying $U(x)$ or $U(\vec{r})$: How far can we go?

For simplicity/clarity, consider 1D TDSE

$$\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + U(x,t)\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t)}$$

- General case: $U(x,\underset{\uparrow\text{time}}{t})$ hard to handle
- Many useful physical situations: $U = U(x)$ only [time does not appear in potential energy function^{+}]

Question: How far can we go in solving TDSE for $U(x)$, without specifying $U(x)$?

+ Many books called $U(x)$ or $U(x,t)$ the "potential". Since U should carry the units of energy, it is clearer to call it the "potential energy function".

Let's go! $U = U(x)$ only (but $U(x)$'s explicit form is not needed)

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + U(x)\,\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t) \qquad \text{(TDSE)}$$

A. Try the method of "<u>Separation of variables</u>"

Try

$$\boxed{\begin{array}{c} \Psi(\underset{\uparrow}{x},\underset{\uparrow}{t}) = \underbrace{\psi(x)}_{\substack{\text{function of}\\ x \text{ only}}} \cdot \underbrace{T(t)}_{\substack{\text{function of}\\ t \text{ only}}} \\ x \text{ and } t \qquad [\text{spatial part}]\cdot[\text{time part}] \end{array}}$$

- Important step
- Will come back to better appreciate what it says
- Can always try this, but useful or not is another business

Plug $\Psi(x,t) = \psi(x)\cdot T(t)$ into TDSE

$$LHS = -\frac{\hbar^2}{2m}\underbrace{\frac{\partial^2}{\partial x^2}}_{}\left[\underbrace{\psi(x)}_{}\cdot T(t)\right] + U(x)\left[\psi(x)\cdot T(t)\right]$$

$$= -\frac{\hbar^2}{2m}T(t)\frac{d^2}{dx^2}\psi(x) + U(x)\,\psi(x)\,T(t)$$

$$RHS = i\hbar\underbrace{\frac{\partial}{\partial t}}_{}\left[\psi(x)\cdot \underbrace{T(t)}_{}\right] = i\hbar\,\psi(x)\frac{d}{dt}T(t)$$

$$TDSE \Rightarrow LHS = RHS = -\frac{\hbar^2}{2m}T(t)\frac{d^2}{dx^2}\psi(x) + U(x)\psi(x)T(t) = i\hbar\,\psi(x)\frac{d}{dt}T(t)$$

Divide by $\Psi = \psi(x)\cdot T(t)$ on both sides:

$$\boxed{-\frac{\hbar^2}{2m}\frac{1}{\psi(x)}\frac{d^2}{dx^2}\psi(x) + U(x) = i\hbar\frac{1}{T(t)}\frac{d}{dt}T(t)}$$

This is TDSE after trying separation of variables

Now, we judge whether separation of variables is useful or not

$$\underbrace{-\frac{\hbar^2}{2m}\frac{1}{\psi(x)}\frac{d^2}{dx^2}\psi(x) + U(x)}_{\text{LHS is a function of } x \text{ ONLY}} = \underbrace{i\hbar\frac{1}{T(t)}\frac{d}{dt}T(t)}_{\text{RHS is a function of } t \text{ ONLY}}$$

Spirit of Separation of Variables

[needed $U=U(x)$ in this step]

Both sides must be equal for all x and all t

This "all x and all t" requirement is the key point of the separation of variables method

- You may think...
  - By luck, there may be $\underbrace{\text{a value of } x \text{ and a value of } t}_{}$ that LHS = RHS

OK. Then keep value of x and change time to another value, then LHS $\neq$ RHS

- Key Point: Only when LHS = E = a constant = RHS, LHS = RHS for all x and t

- Want LHS = RHS for all x and all t
- This is possible *only if*

$$\underbrace{-\frac{\hbar^2}{2m}\frac{1}{\psi(x)}\frac{d^2}{dx^2}\psi(x) + U(x)}_{E} = \underbrace{i\hbar\frac{1}{T(t)}\frac{d}{dt}T(t)}_{E}$$

E ← a constant that does not depend on x & t → E

$\boxed{\text{Same } E}$

"E" is called the "separation constant"

This "same E" is an obvious point often overlooked!

Thus, two "separate" equations arise (but they are related through E)

B. Look at "LHS=E" (the spatial equation)

$\boxed{LHS = E}$ gives

$$\boxed{-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) + U(x)\psi(x) = E\psi(x)}$$

This is the Time-independent Schrödinger Equation (TISE)

[This is the "most important" equation in Quantum Mechanics]

- What is the unit of E?

must be an energy

$$\underbrace{-\frac{\hbar^2}{2m}\frac{1}{\psi(x)}\frac{d^2}{dx^2}\psi(x)}_{\text{Units of } \underbrace{\frac{-\hbar^2}{2m}\frac{1}{(\text{length})^2}}_{\text{energy}}} + \underbrace{U(x)}_{\text{potential energy}} = E$$

$\therefore$ $\boxed{\text{E carries the unit of an energy}}$

or simply $\boxed{\text{E is an energy}}$

[Remark: TISE is analogous to the normal mode equation in classical wave problems]

What does $\frac{-\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) + U(x)\psi(x) = E\psi(x)$ do?

- Specify form of $U(x)$ ⇒ a differential equation to $\underbrace{\text{solve for } E \text{ and } \psi_E(x)}$

$\psi_E(x)$ gives the wavefunction corresponding to energy E

- Schrödinger (1926) applied TISE to hydrogen atom, i.e.

$$\frac{-\hbar^2}{2m}\nabla^2\underbrace{\psi(\vec{r})}_{\substack{\psi(x,y,z)\\ (\text{3D problem})}} - \underbrace{\frac{e^2}{4\pi\varepsilon_0 r}}_{\substack{U(x,y,z)\\ \text{for electron at } \vec{r}=(x,y,z)\\ \text{due to nucleus at } \vec{r}=(0,0,0)}}\psi(\vec{r}) = E\psi(\vec{r})$$

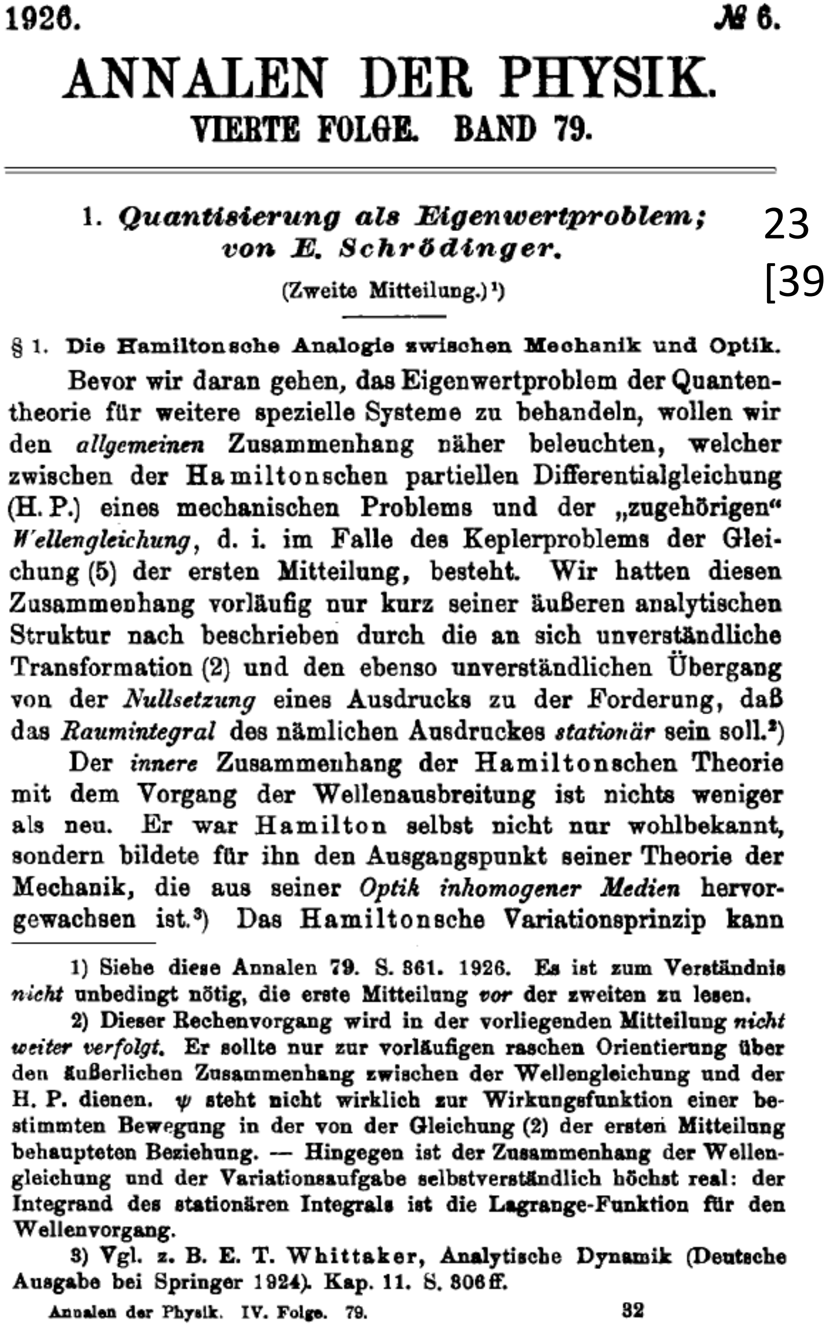
Found $E<0$ solutions at discrete values given by

$$E_n = \underbrace{\frac{-2\pi^2 m e^4}{h^2}}_{-13.6\text{eV}} \cdot \frac{1}{n^2}$$

We will do the H-atom problem later in the course

Erwin Schrodinger (1887 – 1961)
1933 Nobel Prize
(shared Prize with Dirac)

Wrote 2 papers in Annalen der
Physik in 1926
(27 Jan 1926 & 23 Feb 1926)



1926. № 6.

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VIERTE FOLGE. BAND 79.

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1. *Quantisierung als Eigenwertproblem;*
von E. Schrödinger.

(Zweite Mitteilung.)[1]

§ 1. Die Hamiltonsche Analogie zwischen Mechanik und Optik.

Bevor wir daran gehen, das Eigenwertproblem der Quantentheorie für weitere spezielle Systeme zu behandeln, wollen wir den *allgemeinen* Zusammenhang näher beleuchten, welcher zwischen der Hamiltonschen partiellen Differentialgleichung (H. P.) eines mechanischen Problems und der „zugehörigen“ *Wellengleichung*, d. i. im Falle des Keplerproblems der Gleichung (5) der ersten Mitteilung, besteht. Wir hatten diesen Zusammenhang vorläufig nur kurz seiner äußeren analytischen Struktur nach beschrieben durch die an sich unverständliche Transformation (2) und den ebenso unverständlichen Übergang von der *Nullsetzung* eines Ausdrucks zu der Forderung, daß das *Raumintegral* des nämlichen Ausdruckes *stationär* sein soll.[2])

Der *innere* Zusammenhang der Hamiltonschen Theorie mit dem Vorgang der Wellenausbreitung ist nichts weniger als neu. Er war Hamilton selbst nicht nur wohlbekannt, sondern bildete für ihn den Ausgangspunkt seiner Theorie der Mechanik, die aus seiner *Optik inhomogener Medien* hervorgewachsen ist.[3]) Das Hamiltonsche Variationsprinzip kann

---

[1]) Siehe diese Annalen 79. S. 361. 1926. Es ist zum Verständnis *nicht* unbedingt nötig, die erste Mitteilung *vor* der zweiten zu lesen.

[2]) Dieser Rechenvorgang wird in der vorliegenden Mitteilung *nicht weiter verfolgt*. Er sollte nur zur vorläufigen raschen Orientierung über den äußerlichen Zusammenhang zwischen der Wellengleichung und der H. P. dienen. ψ steht nicht wirklich zur Wirkungsfunktion einer bestimmten Bewegung in der von der Gleichung (2) der ersten Mitteilung behaupteten Beziehung. — Hingegen ist der Zusammenhang der Wellengleichung und der Variationsaufgabe selbstverständlich höchst real: der Integrand des stationären Integrals ist die Lagrange-Funktion für den Wellenvorgang.

[3]) Vgl. z. B. E. T. Whittaker, Analytische Dynamik (Deutsche Ausgabe bei Springer 1924). Kap. 11. S. 306ff.

Annalen der Physik. IV. Folge. 79. 32

23 February 1926
[39 pages]

After 22 pages, there came the Time-independent Schrodinger Equation (TISE)

$$\text{div grad}\, \psi + \frac{8\pi^2}{h^2}(E - V)\psi = 0 \qquad (18'')$$

Schrodinger then applied his method to 1D harmonic oscillator (page 26)

$$\frac{d^2\psi}{dq^2} + \frac{8\pi^2}{h^2}(E - 2\pi^2 \nu_0^2 q^2)\psi = 0 \qquad (22)$$

[Why? Recall implications of *Planck's formula* and *Einstein's model of solids*]

Schrodinger solved (22) skillfully (we will do it later) and obtained

$$E_n = \frac{2n+1}{2} h\nu_0 \;; \quad n = 0, 1, 2, 3, \ldots. \qquad (25')$$

$$\therefore \; E_n = nh\nu_0 + \frac{1}{2}h\nu_0$$

as implied by thermal radiation and heat capacities of solids

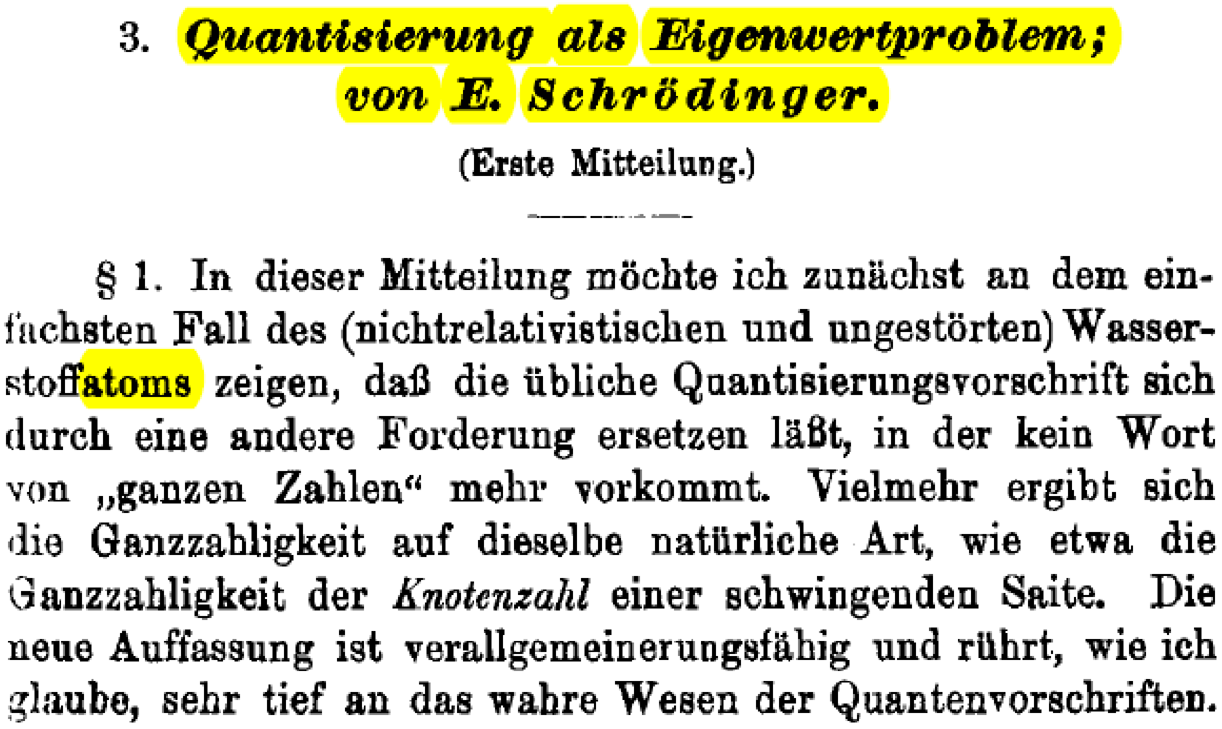
AND

$$\psi_n(q) = e^{-\frac{2\pi^2\nu_0 q^2}{h}} H_n\left(2\pi q \sqrt{\frac{\nu_0}{h}}\right) \qquad (26')$$

Schrodinger then moved on to work out

- the rigid Rotator problem in 2D and 3D
- the 3D oscillator problem

Schrodinger (27 January 1926) in Annalen der Physik (16 pages)



3. *Quantisierung als Eigenwertproblem;*
von E. Schrödinger.

(Erste Mitteilung.)

§ 1. In dieser Mitteilung möchte ich zunächst an dem einfachsten Fall des (nichtrelativistischen und ungestörten) Wasserstoffatoms zeigen, daß die übliche Quantisierungsvorschrift sich durch eine andere Forderung ersetzen läßt, in der kein Wort von „ganzen Zahlen" mehr vorkommt. Vielmehr ergibt sich die Ganzzahligkeit auf dieselbe natürliche Art, wie etwa die Ganzzahligkeit der *Knotenzahl* einer schwingenden Saite. Die neue Auffassung ist verallgemeinerungsfähig und rührt, wie ich glaube, sehr tief an das wahre Wesen der Quantenvorschriften.

Worked on the Hydrogen atom problem (allowed energies are known from spectrum and Bohr's model (1913))

$$\Delta\psi + \frac{2m}{K^2}\left(E + \frac{e^2}{r}\right)\psi = 0$$

After 10 pages of calculations, Schrodinger obtained E<0 solutions for hydrogen with energies given by

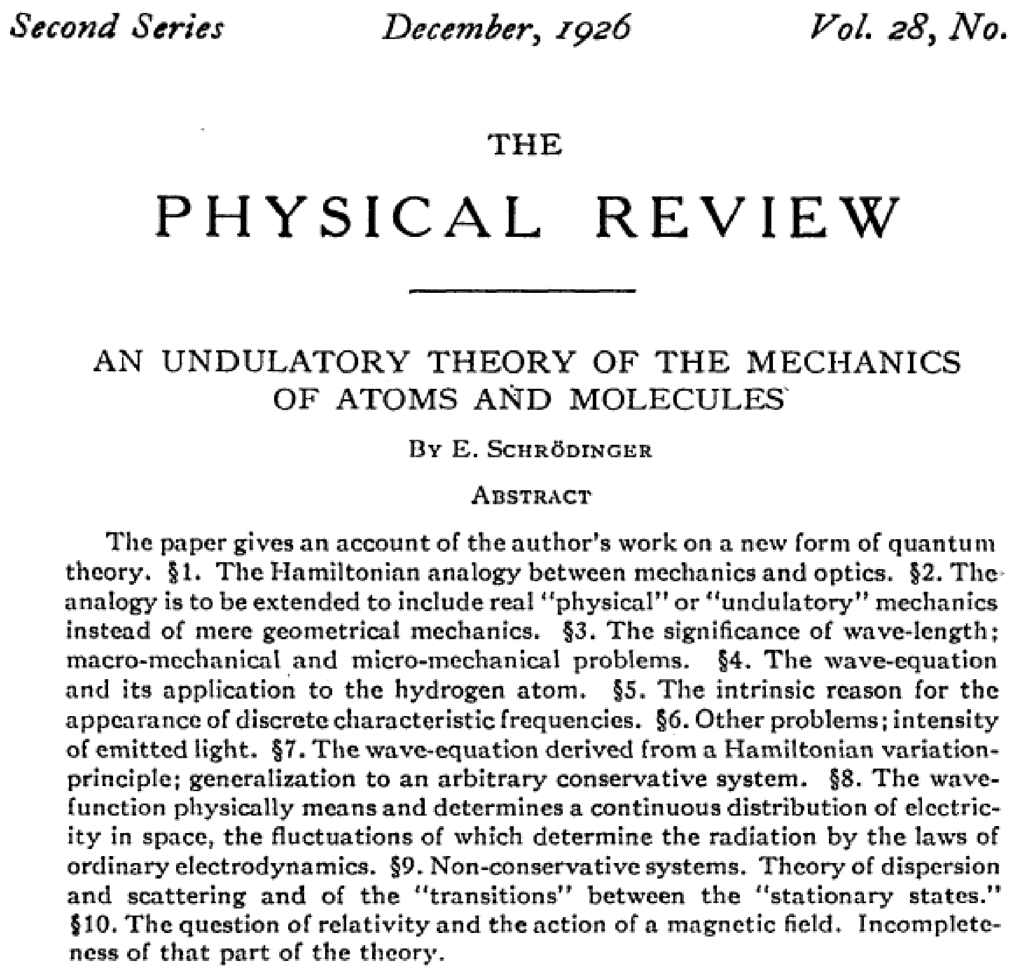
$$-E_l = \frac{me^4}{2K^2 l^2}$$

In modern form,
l is the quantum number n
K is $(\frac{h}{2\pi})$

and the Hydrogen atom wavefunctions

Since the hydrogen atom allowed energies $-\frac{13.6}{n^2}$ (eV) were known (Balmer, Lyman series) at that time, Schrodinger showed explicitly that by solving his equation and requiring well-behaving wavefunctions, *E* in his equation gives the system's (H-atom) allowed energies.

In September 1926, Schrodinger wrote a long paper introducing his theory in English in the *Physical Review* (published by the American Physical Society)



Second Series *December, 1926* *Vol. 28, No. 6*

THE

PHYSICAL REVIEW

AN UNDULATORY THEORY OF THE MECHANICS OF ATOMS AND MOLECULES

BY E. SCHRÖDINGER

ABSTRACT

The paper gives an account of the author's work on a new form of quantum theory. §1. The Hamiltonian analogy between mechanics and optics. §2. The analogy is to be extended to include real "physical" or "undulatory" mechanics instead of mere geometrical mechanics. §3. The significance of wave-length; macro-mechanical and micro-mechanical problems. §4. The wave-equation and its application to the hydrogen atom. §5. The intrinsic reason for the appearance of discrete characteristic frequencies. §6. Other problems; intensity of emitted light. §7. The wave-equation derived from a Hamiltonian variation-principle; generalization to an arbitrary conservative system. §8. The wave-function physically means and determines a continuous distribution of electricity in space, the fluctuations of which determine the radiation by the laws of ordinary electrodynamics. §9. Non-conservative systems. Theory of dispersion and scattering and of the "transitions" between the "stationary states." §10. The question of relativity and the action of a magnetic field. Incompleteness of that part of the theory.

E. Schrodinger, Physical Review 28 (1926) 1049
https://journals.aps.org/pr/abstract/10.1103/PhysRev.28.1049
(accessible from CUHK sites or VPN)

- Schrödinger found that his TISE is:
  - one equation for $\underbrace{\text{many } E\text{'s}}$ and $\underbrace{\text{many } \psi_E(x)}$ [OR $\psi_E(\vec{r})$]

go in a pair

$$\boxed{\begin{array}{ccc} E_1 & \longleftrightarrow & \psi_{E_1}(x) \\ E_2 & \longleftrightarrow & \psi_{E_2}(x) \\ \vdots & & \\ E_n & \longleftrightarrow & \psi_{E_n}(x) \\ \vdots & & \vdots \end{array}}$$

ALL come out from solving TISE

  - Values of E are the allowed energies of a hydrogen atom as deduced from spectrum [Lyman, Balmer, ... series]

∴ Allowed values of E are the allowed energies of the system and $\psi_E(x)$ is the wavefunction of a state of the system with definite energy E

- Key Points

[Solving TISE gives the (many) allowed energies of a quantum system and the wavefunction $\psi_E(x)$ corresponding to a state of definite energy E.

Key Points from Schrodinger's early applications (important concepts):

- TISE is an equation to solve for the wavefunction Ψ ***and** E*
- Not all values of *E* are allowed, only certain values of *E* give well-behaved Ψ
- A well-behaved (QM allowed) Ψ has an accompanying allowed *E*
- The **allowed *E*** values are the **allowed values of energy of the system**

The separation constant *E* happens to give the allowed values of a quantum mechanical problem as specified by U(x)

C. Look at "RHS = E" (the temporal (time) equation)

$\boxed{RHS = E}$ gives $\boxed{i\hbar \frac{d}{dt} T(t) = E\, T(t)}$ ← Governs the time (temporal) part, i.e. how a state of definite energy E evolves in time

$$\frac{dT(t)}{dt} = \frac{E}{i\hbar} T(t) = \frac{-iE}{\hbar} T(t)$$

Asking: "Find T(t) that "$\frac{d}{dt}$" it once gives itself with a prefactor"

By inspection, $\boxed{T(t) = e^{-\frac{iEt}{\hbar}}}$ (could have a constant factor in front)

Physical Meaning

- A state of definite energy E (obtained by solving TISE), thus with wavefunction $\psi_E(x)$, evolves in time as $\psi_E(x)\, e^{-\frac{iEt}{\hbar}}$.

Important QM concept!

More explicit meaning

- Recall TISE $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$ allows many solutions

$$E_1 \leftrightarrow \psi_{E_1}(x) \quad \text{evolves in time as} \quad \psi_{E_1}(x)\, e^{-\frac{iE_1 t}{\hbar}}$$

$$E_2 \leftrightarrow \psi_{E_2}(x) \quad \text{evolves in time as} \quad \psi_{E_2}(x)\, e^{-\frac{iE_2 t}{\hbar}}$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$E_n \leftrightarrow \psi_{E_n}(x) \quad \text{evolves in time as} \quad \psi_{E_n}(x)\, e^{-\frac{iE_n t}{\hbar}}$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

obtained by solving TISE
They are states of definite energy

each with <u>its own</u> time part,
as a result of TDSE
and $U = U(x)$ only

D. Putting Things together: Back to Time-dependent Schrödinger Equation

- Consider a linear combination of $\psi_{E_n}(x)\, e^{-i\frac{E_n t}{\hbar}}$, i.e.

$$\Psi(x,t) = a_1 \psi_{E_1}(x)\, e^{-i\frac{E_1 t}{\hbar}} + a_2 \psi_{E_2}(x)\, e^{-i\frac{E_2 t}{\hbar}} + \cdots + a_n \psi_{E_n}(x)\, e^{-iE_n t/\hbar} + \cdots$$

$$= \sum_{n=1}^{\infty} a_n \psi_{E_n}\, e^{-i\frac{E_n t}{\hbar}}$$ (assuming TISE allows infinitely many solutions)

<u>Important QM concept</u>

Such a combination is a solution to the time-dependent Schrödinger Equation

Here is the proof

Why?

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + U(x)\Psi(x,t) \longleftarrow \text{LHS of TDSE}$$

$$= -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\left(\sum_{n=1}^{\infty} a_n \psi_{E_n}(x) e^{-iE_nt/\hbar}\right) + U(x)\sum_{n=1}^{\infty} a_n \psi_{E_n}(x) e^{-\frac{iE_nt}{\hbar}}$$

$$= \sum_{n=1}^{\infty} a_n e^{-\frac{iE_nt}{\hbar}}\left(\frac{-\hbar^2}{2m}\right)\frac{d^2}{dx^2}\psi_{E_n}(x) + \sum_{n=1}^{\infty} a_n e^{-\frac{iE_nt}{\hbar}} U(x)\psi_{E_n}(x)$$

$$= \sum_{n=1}^{\infty} a_n e^{-\frac{iE_nt}{\hbar}}\left[\frac{-\hbar^2}{2m}\frac{d^2}{dx^2}\psi_{E_n}(x) + U(x)\psi_{E_n}(x)\right]$$

$$= \sum_{n=1}^{\infty} a_n \underbrace{E_n}_{} \psi_{E_n}(x) e^{-\frac{iE_nt}{\hbar}}$$

RHS of TDSE

$$\xrightarrow{} i\hbar\frac{\partial}{\partial t}\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\sum_{n=1}^{\infty} a_n\psi_{E_n}(x)e^{-\frac{iE_nt}{\hbar}} = i\hbar\sum_{n=1}^{\infty} a_n\psi_{E_n}(x)\frac{d}{dt}\left(e^{-\frac{iE_nt}{\hbar}}\right)$$

$$= \sum_{n=1}^{\infty} a_n E_n \psi_{E_n}(x) e^{-\frac{iE_nt}{\hbar}} = \text{LHS}$$

$\therefore$ The combination $\Psi(x,t) = \sum_{n=1}^{\infty} a_n \psi_{E_n}(x) e^{-\frac{iE_nt}{\hbar}}$ satisfies TDSE. Done!

Reminder: The result is more important than the proof!

A quick application: Time evolution of a "specific form"

- Given a system, i.e. $U(x)$ or $U(\vec{r})$

Let's say the state of the system at some time (called) $t=0$ is:

$$\boxed{\Psi(x,t=0) = \Psi(x,0) = a_1\psi_{E_1}(x) + a_2\psi_{E_2}(x) + \cdots + a_n\psi_{E_n}(x) + \cdots} \quad (*)$$

Question: Let $\Psi(x,t=0)$ evolve in time (don't do anything[†] in between), what is $\Psi(x,t)$?

Reasoning: $\psi_{E_n}(x)$ is a solution to TISE, thus a state of energy E_n, it evolves as $\psi_{E_n}(x)\,e^{-\frac{iE_nt}{\hbar}}$. Therefore, each term evolves with its own time part.

Answer:

$$\boxed{\Psi(x,t) = a_1\psi_{E_1}(x)\,e^{-\frac{iE_1t}{\hbar}} + a_2\psi_{E_2}(x)\,e^{-\frac{iE_2t}{\hbar}} + \cdots + a_n\psi_{E_n}(x)\,e^{-\frac{iE_nt}{\hbar}} + \cdots} \quad (+)$$

[†] Meaning: Don't make any measurement, because measurements play a special role in QM and they change the state.

E. General *Initial Value Problems* in Quantum Mechanics

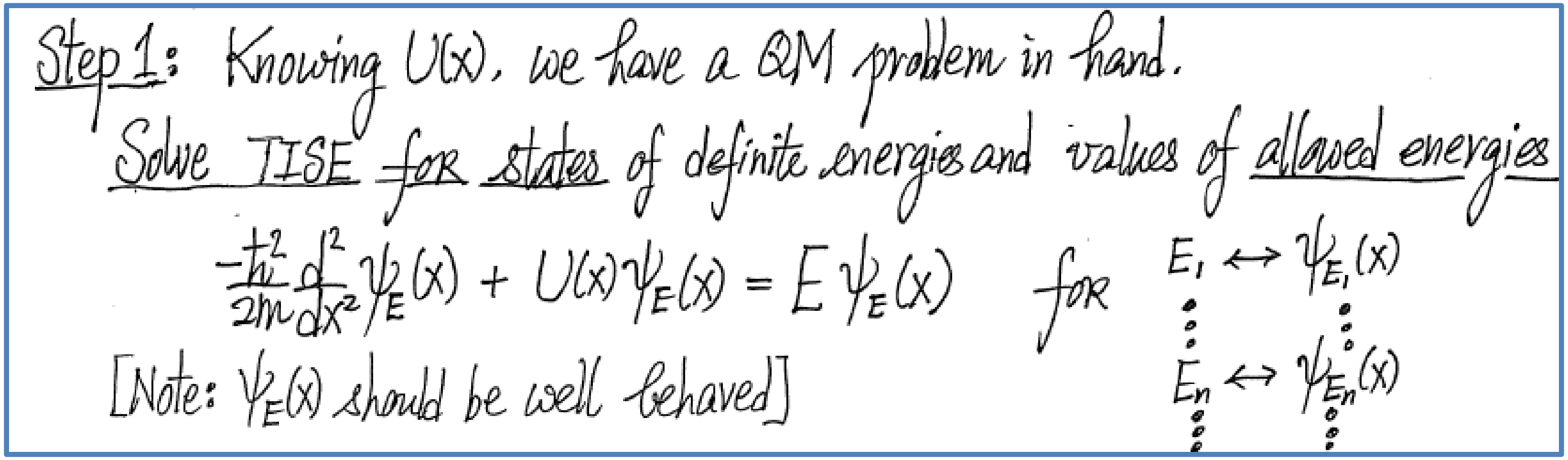
- Need a system, i.e. need to know $U(x)$[†] OR $U(\vec{r})$

Question: The state of the system at $t=0$ is given to be some form called $\Phi(x,t=0) = \Phi(x)$. What is the state $\Phi(x,t)$ at time t?

Putting the results together, we have the following *3-step recipe*

[†] If $U = U(x,t)$, the problem is harder. The recipe here does not work for $U(x,t)$. However, the time-evolution is still governed by TDSE.

Step 1: Solve the time-independent Schrodinger Equation



Step 1: Knowing $U(x)$, we have a QM problem in hand.

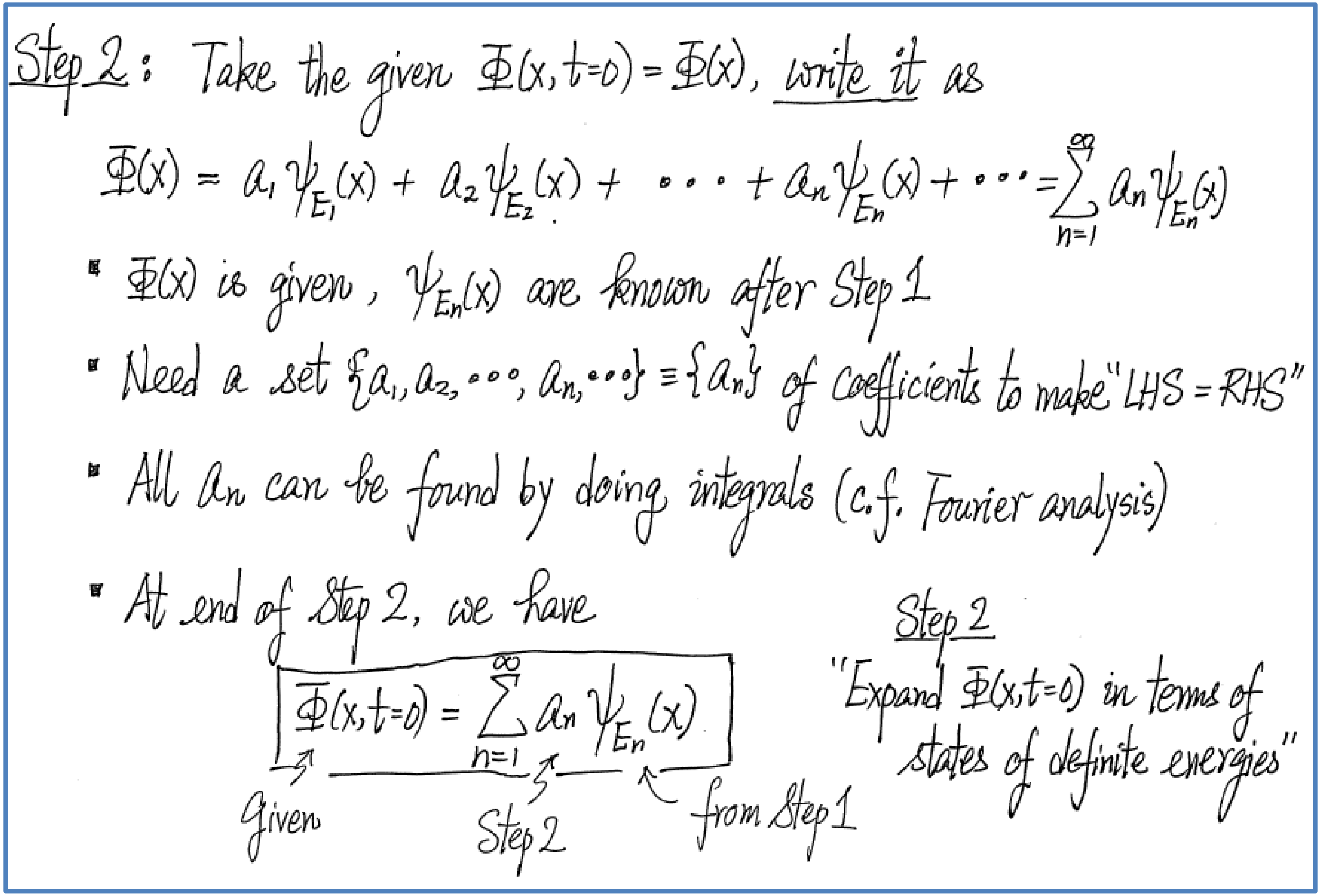
Solve TISE for states of definite energies and values of allowed energies

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi_E(x) + U(x)\psi_E(x) = E\,\psi_E(x) \quad \text{for} \quad \begin{array}{ccc} E_1 & \leftrightarrow & \psi_{E_1}(x) \\ \vdots & & \vdots \\ E_n & \leftrightarrow & \psi_{E_n}(x) \\ \vdots & & \vdots \end{array}$$

[Note: $\psi_E(x)$ should be well behaved]

Remark: This is why we will work out many TISE problems in 1D, 2D, 3D

Step 2: Expand given initial wavefunction in terms of states of definite energies



Step 2: Take the given $\Phi(x,t=0)=\Phi(x)$, write it as

$$\Phi(x) = a_1\psi_{E_1}(x) + a_2\psi_{E_2}(x) + \cdots + a_n\psi_{E_n}(x) + \cdots = \sum_{n=1}^{\infty} a_n\psi_{E_n}(x)$$

- $\Phi(x)$ is given, $\psi_{E_n}(x)$ are known after Step 1
- Need a set $\{a_1, a_2, \cdots, a_n, \cdots\} \equiv \{a_n\}$ of coefficients to make "LHS = RHS"
- All a_n can be found by doing integrals (c.f. Fourier analysis)
- At end of Step 2, we have

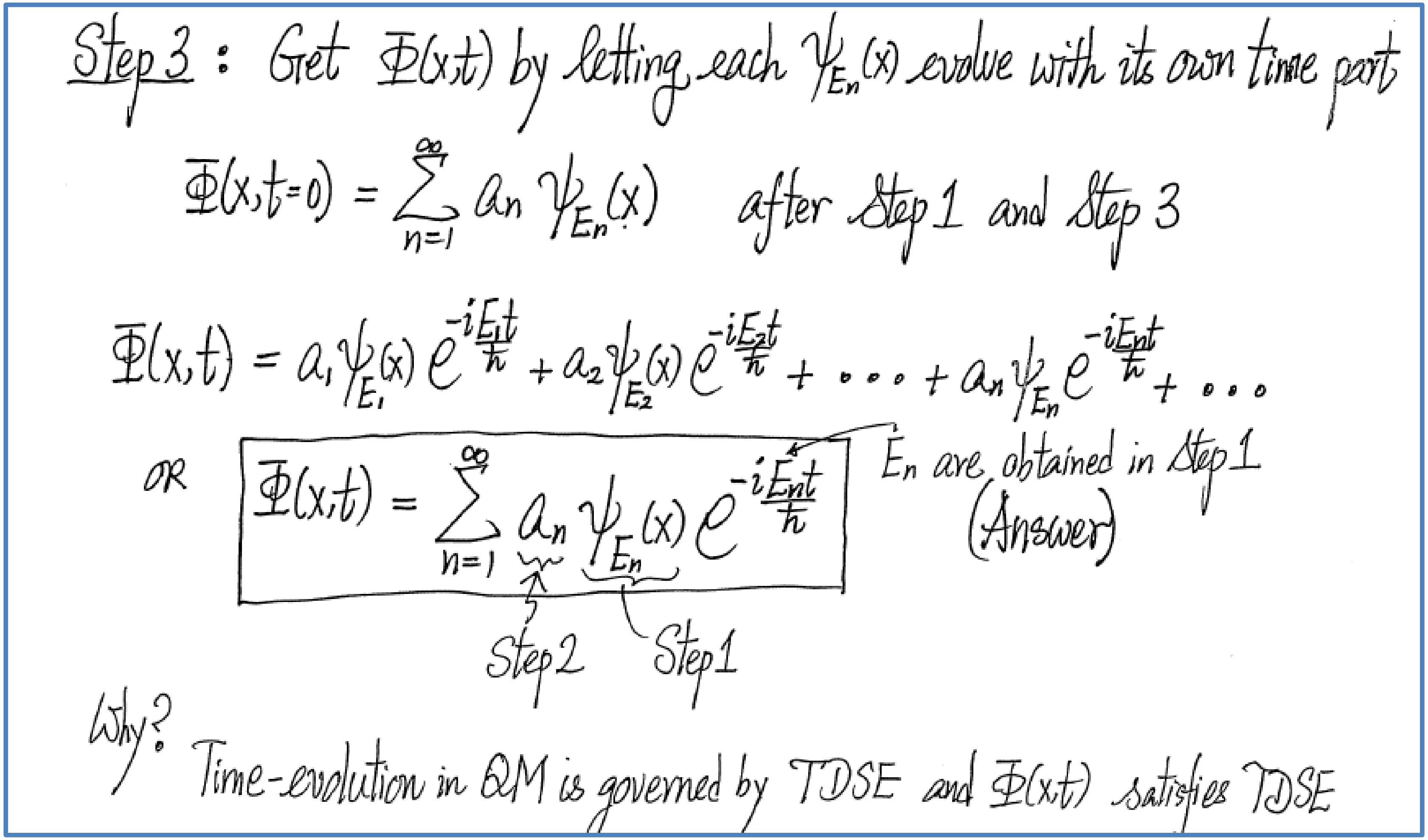
$$\boxed{\underbrace{\Phi(x,t=0)}_{\text{given}} = \sum_{n=1}^{\infty} \underbrace{a_n}_{\text{Step 2}} \underbrace{\psi_{E_n}(x)}_{\text{from Step 1}}}$$

Step 2

"Expand $\Phi(x,t=0)$ in terms of states of definite energies"

Remark: See Sample Question SQ10 for an example

Step 3: Let each definite energy state evolve with its own time part



Step 3 : Get $\Phi(x,t)$ by letting each $\psi_{E_n}(x)$ evolve with its own time part

$$\Phi(x,t=0) = \sum_{n=1}^{\infty} a_n \psi_{E_n}(x)$$

after Step 1 and Step 3

$$\Phi(x,t) = a_1\psi_{E_1}(x)\,e^{\frac{-iE_1t}{\hbar}} + a_2\psi_{E_2}(x)\,e^{\frac{-iE_2t}{\hbar}} + \cdots + a_n\psi_{E_n}\,e^{\frac{-iE_nt}{\hbar}} + \cdots$$

OR

$$\boxed{\Phi(x,t) = \sum_{n=1}^{\infty} \underbrace{a_n}_{\text{Step2}}\,\underbrace{\psi_{E_n}(x)}_{\text{Step1}}\,e^{\frac{-iE_nt}{\hbar}}}$$

E_n are obtained in Step 1

(Answer)

Why? Time-evolution in QM is governed by TDSE and $\Phi(x,t)$ satisfies TDSE

Initial-value problems are a big part of QM. Steps 1,2,3 solve the problems for $U = U(x)$ or $U(\vec{r})$ [U with no explicit time in it]

All Done (without knowing the explicit form of *U(x)*)!

Remark: For those who have studied the classical wave equation (e.g. a string fixed at its two ends), the steps in solving initial value problems starting from TDSE are exactly the same as those starting from the classical wave equation. See Problem Set 1.

Check Point/Summary

- How does "separation of variables" work?
- From TDSE ↗ TISE + ↘ temporal equation } how?
- What does TISE do?

  Solving TISE, what does it mean? Solving for what?
- How does a state of definite energy evolve in time?
- How does a state $\Psi(x,t=0) = a_1\psi_{E_1}(x) + a_2\psi_{E_2}(x) + \cdots + a_n\psi_{E_n}(x) + \cdots$ evolve in time?

- FOR a system defined by $U(x)$ OR $U(\vec{r})$, what are the steps for solving the initial value problem that $\Psi(x,t=0) = \Psi(x,0) = \Phi(x)$?

- Write down TDSE and TISE for standard problems

Think through these check-point questions and write down your answers quietly while explaining the ideas to yourself is an effective way to check on your understanding.